MAT 335
Practice Problems

1. In old growth forests of Douglas fir, the spotted owl feasts mainly upon flying squirrels. If $\mathbf{p}_k = \begin{bmatrix} \mathbf{o}_k \\ \mathbf{s}_k \end{bmatrix}$ is a vector describing the population of owls and squirrels (in thousands) at the kth month and $\mathbf{p}_k = A\mathbf{p}_{k-1}$, with
$A = \begin{bmatrix} .4 & .3 \\ -.4 & 1.2 \end{bmatrix}$

   a. Write a general form for $\mathbf{p}_k$ in terms of eigenvalues and eigenvectors of A.

   b. Describe what happens to the populations in the long run.

2. Suppose $\mathbf{c} = \begin{bmatrix} \text{station A} \\ \text{station B} \\ \text{station C} \end{bmatrix}$ and $T = \begin{bmatrix} .83 & .15 & .35 \\ .07 & .73 & .25 \\ .10 & .12 & .40 \end{bmatrix}$ describes weekly transitions of rental cars from one station to another:
(*i.e.*: all cars are rented from one of the 3 stations are returned to one of those 3)

   a. Diagonalize the matrix T (Find P and D such that $T = PDP^{-1}$).

   b. Describe what happens to the distribution of cars in the long run assuming T continues to describe transitions between stations.

   c. if the initial distribution of cars is $\mathbf{c}_0 = \begin{bmatrix} 400 \\ 200 \\ 200 \end{bmatrix}$, what is the limiting distribution?

3. Suppose the population dynamics, yearly, for females in a herd of bison are as follows:
i) An average of 42 female calves are born each year per 100 adult females
ii) Each year, 60% of female calves survive to become yearlings
iii) Each year, 75% of female yearlings survive to become adults
iv) Each year, 95% of female adults survive to the next year

   a. Define a population vector, **p**, and write a yearly transition matrix, T, for this population.

   b. Write a general expression for $\mathbf{p}_k$ in terms of eigenvalues and eigenvectors of T

   c. Show that the herd is growing, describe the growth rate and the relative proportion of calves, yearlings and adults in the female population in the long run.